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Classification of simple C^*-algebra of generalized tracial rank 1.

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(Joint with Huaxin Lin and Zhuang Niu)

Elliott project: classify all simple separable nuclear C^*-algebras via K-theoretical invariant

Elliott invariant (unital case)

$(K_0(A), K_0(A)_+, [1_A], K_1(A), TA, \langle\,,\,\rangle)$

$K_0(A) = \{[p]-[q],\ p, q \in M_n(A) \text{ projections}\}/\sim$

$K_0(A)_+ \subset K_0(A)$ is the semigroup generated by $[p] \sim 0 \in K_0(A)$.

$[1_A] \in K_0(A)$ (unit of A)

TA — space of tracial states

$\langle\ \rangle : TA \times K_0A \longrightarrow \mathbb{R}$

$\langle \tau, [p] \rangle = \sum_{i=1}^{n} \tau(p_{ii})$

Weakly unperforated property (A simple)

$K_0(A)$ has w.u.p : if ~~for any~~ $x \in K_0(A)$

$\tau(x) > 0$ for all $\tau \in TA$

$\Longrightarrow x \in K_0(A)_+$

Subhomogeneous algebra: A is called subhomoge
if there is $n \in \mathbb{N}$, such that all irredu
representation of A is of finite dimensiona
and has ~~dimension~~ dim $\leq n$.

ASH algebra: $A_1 \to A_2 \to \cdots \to A$,
where A_i are subhomogeneous algebras.

AH algebras: if $A_i = P_{n,i} M_{n,i} C(X_{n,i}) P_{n,i}$

Rördam: ~~give~~ Counter example of Elliott Conjectur

Toms: Construct counter example in class of AH

Conjecture: All simple sep. nuclear stably finite C^*-algebra are ASH algebras.

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Let $\mathcal{Z}$ be Jiang-Su alg, infinite dimensional simple C^*-algebras with
$$Ell(\mathcal{Z}) = Ell(\mathbb{C}).$$

~~G-Jiang $\forall$ simple~~

~~$\forall$ unital simple A, $A \otimes \mathcal{Z}$ has weakly w.u.p.~~

~~(G-Jiang-Su) constructed AH algebra A such that~~

A is called $\mathcal{Z}$-stable if $A \cong A \otimes \mathcal{Z}$.

(G-Jiang-su): $\exists$ simple AH algebra A, which is not $\mathcal{Z}$-s

Fact 1. For simple sep. A, $A \otimes \mathcal{Z}$ has w.u. ~~For simple stably finite weakly unperforated~~

Fact 2. If A has w.u.p. then
$$Ell(A) \cong Ell(A \otimes \mathcal{Z}).$$

Tom constructed AH alg. with w.u.p. and
$$A \not\cong A \otimes \mathcal{Z}.$$

Classification in term of Elliott invariant

Can hold at best ~~at the level~~ for of $\mathcal{Z}$-stable C^*-algebra

~~For AH algebra (ASH algebra, by ...)~~

Slow dimension growth.

let $l(A_n)$ = minimal dimension of irreducible representation of A_n

$$\lim_{n\to\infty} \frac{\dim sp(A_n)}{l(A_n)} = 0$$

A ASH, $A \cong A \otimes \mathcal{Z} \iff A$ has s.d.g.

Elliott-G-Li: If A and B are simple AH algebra with (very) slow dimension growth, then

$A \cong B \iff Ell(A) \cong Ell(B)$

TAI (Lin)

Important ingredient Decomposition theorem (Gong)

If A is as above, then A satisfy the following condition $\forall$ finite set $F \subset A$, $\varepsilon > 0$, ~~and~~

$\forall\, b \in A_+$

and $L \in \mathbb{N}$, $\exists$ a subalgebra

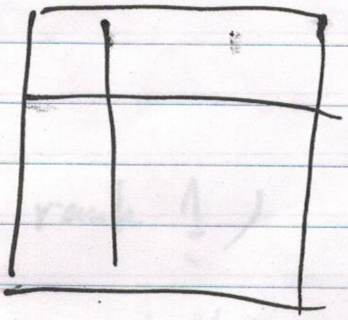
$B = \bigoplus_{i=1}^{n} M_{k_i}(C[0,1]) \subset A$ with the following property (let $1_B = p$)

(1) ~~$[p, \oplus]$~~ $\|pf - fp\| < \varepsilon \quad \forall f \in$ ~~F~~

(2) $d(pfp, B) < \varepsilon \quad \forall f \in F$

(3) $N[1-p] < [p] \in K_0(A)$

$1-p$ ~~equiva~~ $\lesssim b$ ~~$1-p \sim q \in bAb$~~ $1-p \sim q \in bAb$.



If $B = \bigoplus_{i=1}^{n} M_{k_i}(\mathbb{C})$, then A is called TAF.

For general unital simple ASH, it is possible A ~~If we want A to be~~ has no proper projection. ~~$TA\mathcal{C}$, then $\mathcal{C}$~~ ~~will be as~~

$A \in TA\mathcal{C} \Rightarrow A \in \mathcal{C}$.

Winter-Lin If $A \otimes UHF$ is TAF, with an technical condition, then $A \otimes \mathcal{Z}$ can be classify.

removed by Huaxin Lin.

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~~(Lin)~~
~~Kuadin~~
rationally TA$\mathcal{E}$ if $A \otimes M_{\mathbb{Q}}$ is TA$\mathcal{E}$.

~~Theorem~~ Theorem (Lin) if both A an. B are rationally TAI, then

$A \otimes Z \cong B \otimes Z$ if and only if

$Ell(A \otimes Z) \cong Ell(B \otimes Z)$

$\exists$ many simple ASH algebra which is not rationally TAI.

TA$\mathcal{E}\mathcal{J}$ (generalized tracial rank 1)

$A \in \mathcal{E}\mathcal{J}$ is described as below

let $F_1\ F_2$ be f.d. alg.

$\alpha_0, \alpha_1 : F_1 \to F_2$

$$A = \{ (f,a) \in C([0,1], F_2) \oplus F_1 \mid \alpha_0(a) = f(0),\ \alpha_1(a) = f(1) \}$$

Assume $K_1(A) = 0$ then $A \in \mathcal{E}\mathcal{J}_0$

~~Theorem~~
Thm (G-Lin-Niu) For simple nuclear separable C^*-algebra A and B (with UCT)

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If $A\otimes M_{\mathbb{Q}}, B\otimes M_{\mathbb{Q}} \in TAEJ_0$ and

$Ell(A\otimes \mathcal{Z}) \cong Ell(B\otimes \mathcal{Z})$, then $A\otimes \mathcal{Z} \cong B\otimes \mathcal{Z}$

Next goal: ~~Prove~~ Question: are all unital simple stably finite separable nuclear C^*-algebras are rationally $TAEJ$ (At least ~~prove the theorem~~ for ASH algebras)

Remark: For any ~~ADE~~ unital simple separable, stable fi[nite] nuclear C^*-algebra A, $\exists$ B with $B\otimes M_{\mathbb{Q}} \in T$[AEJ] such that $Ell(A\otimes \mathcal{Z}) \cong Ell(B\otimes \mathcal{Z})$.

If Elliott conjecture holds ~~, then~~ all $\mathcal{Z}$-stable algebra, then answer to the question is Y[es]